Introduction of a 3D Particle-In-Cell (PIC) method in a Discontinuous Galerkin electromagnetic model. Application to high power microwave sources (NADEGE project)

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Vlasov-Maxwell equations

Maxwell equations

- Find $(\mathbf{E},\mathbf{H}) : \Omega\times]0,T[\to \mathbb{R}^3 \times \mathbb{R}^3$ such that :

$$\begin{cases} \varepsilon\dfrac{\partial \mathbf{E}}{\partial t} - \nabla\times\mathbf{H} + \mathbf{J} = 0 \text{ on } \Omega \\ \mu\dfrac{\partial \mathbf{H}}{\partial t} + \nabla\times\mathbf{E} = 0 \text{ on } \Omega \\ \mathbf{E}\times\mathbf{n}(\mathbf{x}) = 0 \text{ on } \partial\Omega \\ \nabla\cdot\mathbf{D} = \rho \\ \nabla\cdot\mathbf{B} = 0 \\ \mathbf{E}(\mathbf{x},0) = \mathbf{E}_0(\mathbf{x}) \text{ and } \mathbf{H}(\mathbf{x},0) = \mathbf{H}_0(\mathbf{x}) \text{ on } \Omega \end{cases} \quad (1)$$

- Three-dimentional computational domain Ω.
- $\mathbf{E}$ is the electric field and $\mathbf{H}$ the magnetic field with $\mathbf{E}_0$ and $\mathbf{H}_0$ initial conditions.
- ε, μ and σ denote, respectively, the permittivity, the permeability and the conductivity of the medium.
- $\mathbf{J}(\mathbf{x},t)$ and $\rho(\mathbf{x},t)$ represent electric current and charge densities.

Vlasov equation

- Evolution of the particle distribution function $\mathbf{f}$ in a collisionless plasma :

$$\frac{\partial \mathbf{f}}{\partial t} + \mathbf{v}.\nabla\mathbf{f} + \frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}).\frac{\partial \mathbf{f}}{\partial \mathbf{v}} = 0 \quad (2)$$

This equation is written for each species of particles.

- Invariance of this distribution in time along trajectories submitted to electromagnetic fields.
- Positions and velocities of particles $(\mathbf{x},\mathbf{v})$ are solutions of the characteristics of the Vlasov equation (motion equations) :

$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{v} \\ \frac{d\mathbf{v}}{dt} = \frac{q}{m}[\mathbf{E} + \mathbf{v}\wedge\mathbf{B}] \end{cases} \quad (3)$$

Coupling of equations

- Charge conservation equation :

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\mathbf{J} = 0 \quad (4)$$

- Numerical problem : the discrete charge conservation equation is not satisfied.
- Solution : introduction of a corrector term on electric field E at each time step.
- Choice of a purely hyperbolic correction (appropriated for our schemes)

$$\begin{cases} \varepsilon\dfrac{\partial \mathbf{E}}{\partial t} - \nabla\times\mathbf{H} + \mathbf{J} + \chi\nabla\phi = 0 \\ \mu\dfrac{\partial \mathbf{H}}{\partial t} + \nabla\times\mathbf{E} = 0 \\ \mu\dfrac{\partial \phi}{\partial t} + \chi\nabla\cdot\mathbf{E} = \chi\dfrac{\rho}{\varepsilon_0} \\ \nabla\cdot\mathbf{B} = 0 \end{cases} \quad (5)$$

Discontinuous Galerkin formulation

- Three-dimensional computational domain $\Omega = \cup_{i=1}^{N} K_i$, $(K_i)_{i=1,\cdots,N}$ hexahedra.
- On each cell K, Maxwell equations with hyperbolic correction are rewritten :

$$\begin{cases} \varepsilon_0\dfrac{\partial \mathbf{E}}{\partial t} - \nabla\times\mathbf{H} + \chi\nabla\phi + \mathbf{J} + \alpha[\mathbf{H}\times\mathbf{n}] + \beta[\mathbf{n}\times(\mathbf{E}\times\mathbf{n})] + \tau\chi[\phi\mathbf{n}] = 0 \\ \mu_0\dfrac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} + \delta[\mathbf{E}\times\mathbf{n}] + \gamma[\mathbf{n}\times(\mathbf{H}\times n)] = 0 \\ \mu_0\dfrac{\phi}{\partial t} - \chi\dfrac{\rho}{\varepsilon_0} + \chi\nabla\cdot\mathbf{E} + \eta\chi[\mathbf{E}\cdot\mathbf{n}] - \theta[\phi] = 0 \end{cases} \quad (6)$$

- Conditions : $1 + \alpha - \gamma = 0$, β, $\delta \geqslant 0$ and $1 + \tau + \eta = 0$.
- Boundary conditions for the corrective terms :

$$\begin{matrix} \eta = -1,\ \theta = 1,\ \tau = 0 \text{ (absorbing boundary conditions)} \\ \eta = \theta = 0,\ \tau = -1 \text{ (metallic boundary conditions)} \end{matrix} \quad (7)$$

- $(\mathbf{E},\mathbf{H}) \in (V_r)^2$, $V_r = \{\mathbf{v} \in [L^2(\Omega)]^3 : \forall K \in T, DF_K^{*}\mathbf{v}_{|K} \circ F_K \in [Q_r(K)]^3\}$.
- Elements of the mesh : a conform mapping F_K such that $F_K(\hat{K}) = K$.
- For a spatial approximation order r, $(r+1)^3$ degrees of freedom located at the Gauss quadrature points and 3 polynomial basis functions $\hat{\varphi}$ at each degree.
- Degrees of freedom and basis functions φ, on each element K : $\varphi\circ F_K(\hat{\mathbf{x}}) = DF_K^{*-1}\hat{\varphi}(\hat{\mathbf{x}})$, with DF_K the Jacobian matrix.

Particle-In-Cell method

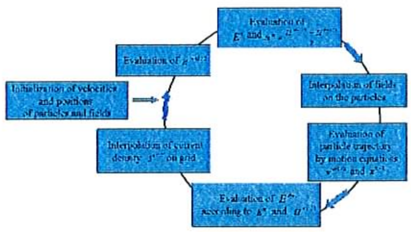


FIGURE 1: Principle of 3D PIC method.

- Current density $\mathbf{J}$ and charge density ρ spawn by charged particles :

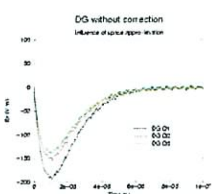
$$\begin{aligned} \rho(\mathbf{x},t) &= \sum_s q_s \int_{\mathbb{R}^3} f_s(\mathbf{x},\mathbf{v},t)d\mathbf{p} \\ \mathbf{J}_s(\mathbf{x},t) &= \sum_s q_s \int_{\mathbb{R}^3} \mathbf{p} f_s(\mathbf{x},\mathbf{v},t)d\mathbf{p} \end{aligned} \quad (8)$$

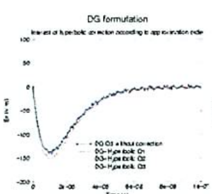
- Interpolation of electromagnetic fields on the particles and particles on mesh with Nearest Grid Point (NGP) method.

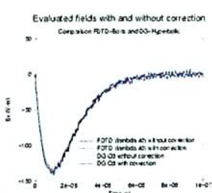
Numerical results

Comparison : DG-Hyperbolic and FDTD-Boris

- Boris' method (elliptic-hyperbolic correction)
- Geometry : perfectly metallic cubic cavity
- Introduction of current and charge densities in the cavity

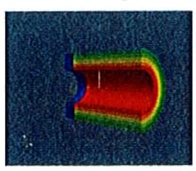


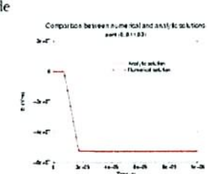




Coaxial line: $R_{ext} = 0.2m$, $R_{int} = 0.1m$, $L = 0.6m$ and $V_{peak} = 4MV$

- Source : Transverse ElectroMagnetic (TEM) mode





- Field emission (breakdown field=2.5e7V/m)