# Numerical methods for fluid and kinetic plasma models in the quasineutral limit

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## Context and Motivations

### Pasma processes main caracteristics

- Plasma core mainly quasineutral ;
- Most of the physics explained by non quasi-neutral localized phenomena (POS operating, instabilities of the tokamak plasma edge, . . .) ;
- Time dependant interfaces between quasineutral and non quasi neutral areas.

### Main stream numerical methods overview

- Asymptotic models derivation ($\mathrm{P}^0$ for $\lambda = \lambda_D/L \ll 1$) to built efficient numerical methods.
- Use of the non quasi-neutral model ($\mathrm{P}^\lambda$ for $\lambda = \lambda_D/L = \mathcal{O}(1)$) in regions where the limit regime is not valid. (Opening of the POS, instabilities of the tokamak plasma edge, . . .) ;
- Coupling strategy with interface tracking procedure.

### Breakthrough strategy : Asymptotic-Preserving methods

- Numerical scheme consistant with $\mathrm{P}^\lambda$ for $\lambda = \mathcal{O}(1)$ ;
- Numerical schema consistant with $\mathrm{P}^0$ for $\lambda \ll 1$ ;
- Inconditionnal stability with respect to $\lambda$.

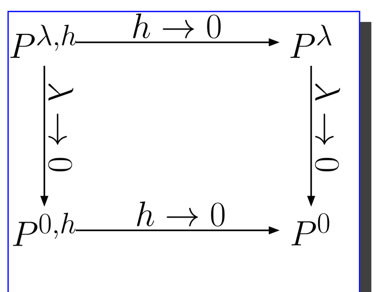



FIGURE 1: AP-schemes consistancy properties.

(a) Z machine. (b) Plasma Opening switch.

FIGURE 2: X-ray generator (Z machine at Sandia Lab, University of texas). Plasma Opening Switch used to compress the power (a). A POS is used to transmit a high power current a compress the pulse(b) : the impedance of the device is rapidly increased thanks to the interaction of a plasma with an electromagnetic field. During this process very localized, non quasi-neutral, phenomena are determinant.

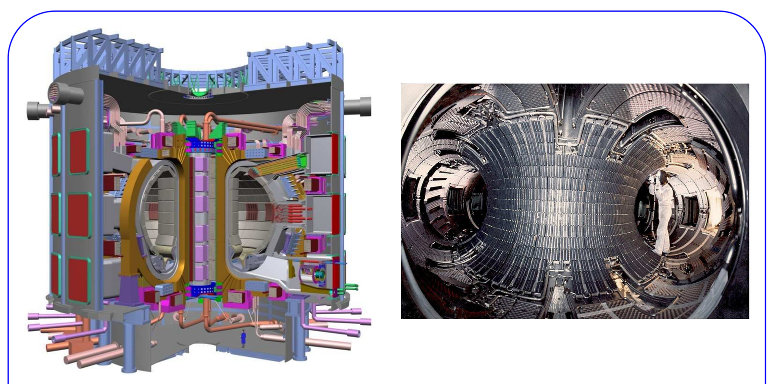


FIGURE 3: Magnetically confined fusion plasma : ITER experiment schematic representation. The simulation of the plasma core as well as the plasma edge is one of the most important challenge for modern numerical tools.

## Asymptotic preserving schemes

### AP-scheme derivation overview : the Euler-Poisson system near quasineutrality

- Definition of the model for the standard regime ($\mathrm{P}^\lambda$)

$$(\mathrm{P}^\lambda) \quad \begin{cases} \partial_t n + \nabla \cdot q = 0, \\ \partial_t q + \nabla \left( \dfrac{q \otimes q}{n} \right) + \nabla p(n) = n \nabla \phi, \\ \lambda^2 \Delta \phi = n - 1, \end{cases}$$

- Investigation of the asymptotic model $P^0$

$$(\mathrm{P}^0) \quad \begin{cases} \nabla \cdot q = 0, \\ \partial_t q + \nabla \left( q \otimes q \right) + p(n) = \nabla \phi, \\ n = 1 \,, \ \text{continuity eq. :} \ \nabla \cdot (n \nabla \phi) = \nabla^2 : \left( \dfrac{q \otimes q}{n} + p(n) \right) \end{cases}$$

- Reformulation : Derive a system of equations equivalent to $P^\lambda$ in which the limit $\lambda \to 0$ is regular : Reformulated Poisson equation

$$\lambda^2 \partial^2_{tt}(-\Delta \phi) - \nabla \cdot (n \nabla \phi) = -\nabla^2 : \left( \frac{q \otimes q}{n} + p(n) \right) .$$

This equation does not degenerate in the limit $\lambda \to 0$. It is equivalent to the Poisson equation under simple assumptions on the initial data.

### AP-schemes for plasmas near quasineutrality : main achievements

- Fluid models : Euler-Poisson System analysis [DLV08] and different AP formulations and 2D numerical bi-fluid simulations [CDV07a, CDV07b]. One dimensional bi-fluid Euler Maxwell system numerical investigations [DDS].
- Kinetic models : Different Asymptotic preserving formulations for the Vlasov-Poisson system [DDN06, DDNSV10]. Ongoing work for the Vlasov Maxwell systems [Doyen].

## Numerical simulations (Fluid models)

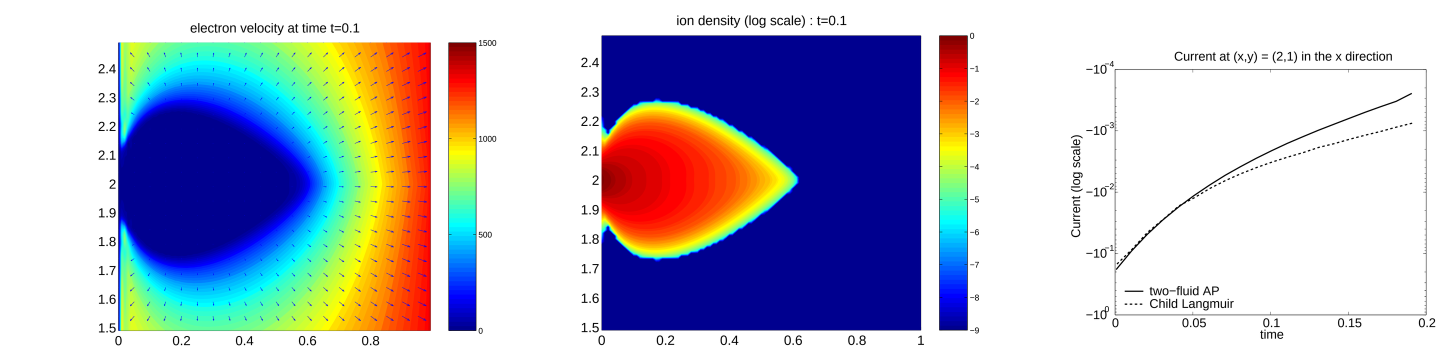



FIGURE 4: Euler-Poisson 2D bluifluid simulation of a plasma slab expansion between two electrons. Computations carried out with an AP-scheme and $10 \le \lambda_D/\Delta x \le 10^{-2}$ and a mass ratio $m_i/m_e = 10^{-4}$. Left (Middle) : electronic speed ; (ionic density) as a function of space (t=0.1 s). Right : time evolution of the flowing current, comparison with Child-Langmuir Law.

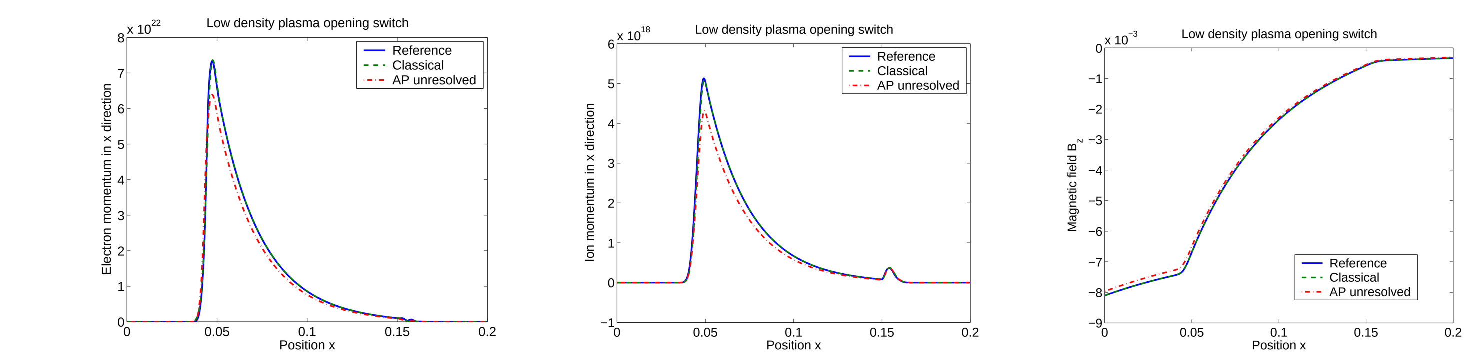



FIGURE 5: Plasma Opening Switch simulation with the 1D bifluid Euler-Maxwell system. Classical scheme compared to an AP-Scheme with either $\Delta t \cdot \omega_{pe} = 1/20$ and $\lambda_D/\Delta x = 5$ (resolved) or $\Delta t \cdot \omega_{pe} = 3$ and $\lambda_D/\Delta x = 1/4$ (unresolved). Left(Middle) : electronic (ionic) momentum ; Right : Magnetic field.

## Numerical simulations (Kinetic models)

### Plasma slab expansion : physical and numerical settings

- Maxwellian distribution with $n_{i0}$ and $n_{e0}$ (see Figure 6) and $T_{e0} = 10^3 T_{i0}$.
- Electron to ion mass ratio $\varepsilon = 1836$.
- Size of the domain $A = 10^3 \lambda$ , size of the half slab $L/2 = 20\lambda$.
- Simulation time : $t = 30\omega_i^{-1} = \sqrt{\varepsilon}\omega$.

| Computation | Resolved | Under res. | Poor res. |
|---|---|---|---|
| $\Delta t$ | $0.05\,\omega^{-1}$ | $3\,\omega^{-1}$ | $3\,\omega^{-1}$ |
| $\Delta x$ | $0.2\lambda$ | $4\lambda$ | $4\lambda$ |
| Nb. of particles | $5 \times 10^6$ | $5 \times 10^6$ | $2.5 \times 10^5$ |

TABLE 1: Computation caracteristics : Resolved, Under resolved and Poor resolution numerical parameters.

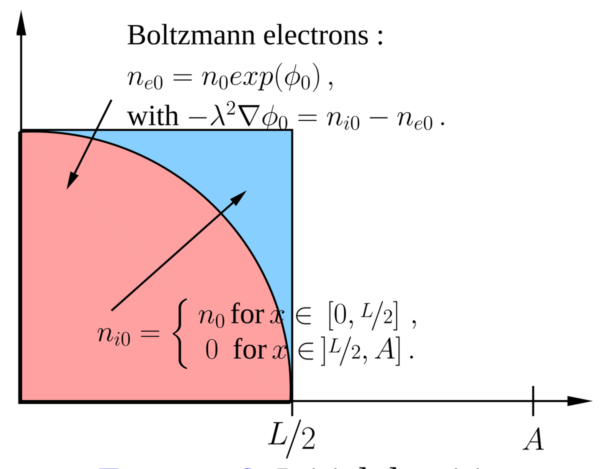
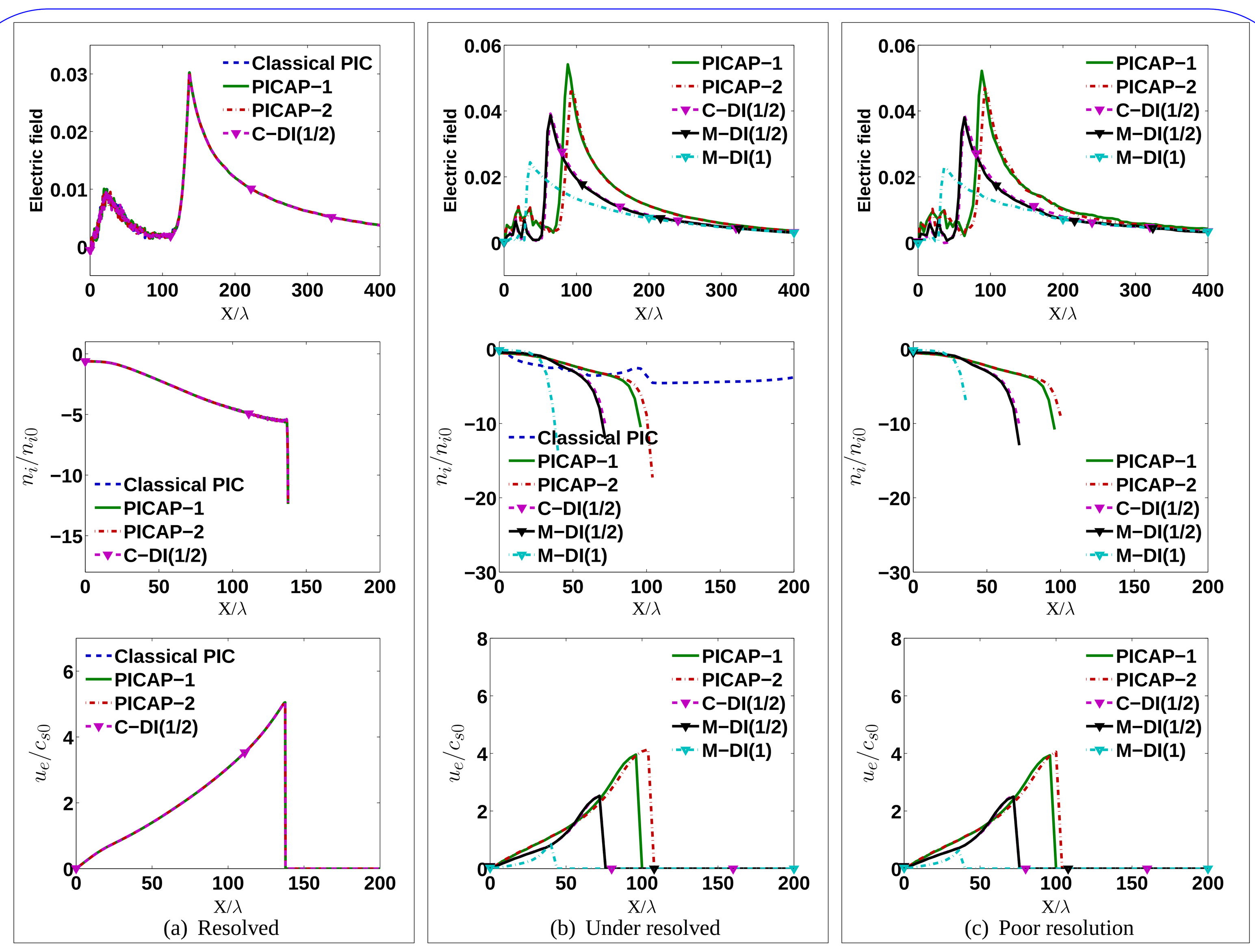



FIGURE 6: Initial densities




(a) Resolved (b) Under resolved (c) Poor resolution

FIGURE 7: Plasma expansion after $t = 30\omega_i^{-1}$ simulated by a bi-fluid 1D Vlasov-Poisson model and using Particle-In-Cell methods. Electric field (top) as plasma density (middle) and electron mean velocity (bottom) as a function of space. Comparisons of a classical explicit scheme, different direct implicit schemes and two Asymptotic-Preserving methods [DDN06, DDNSV10] with different resolutions as detailled in table 1.

## References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## Acknowledgments